

# Symmetry-breaking defects and textures in quantum orders

arXiv:2211.13207

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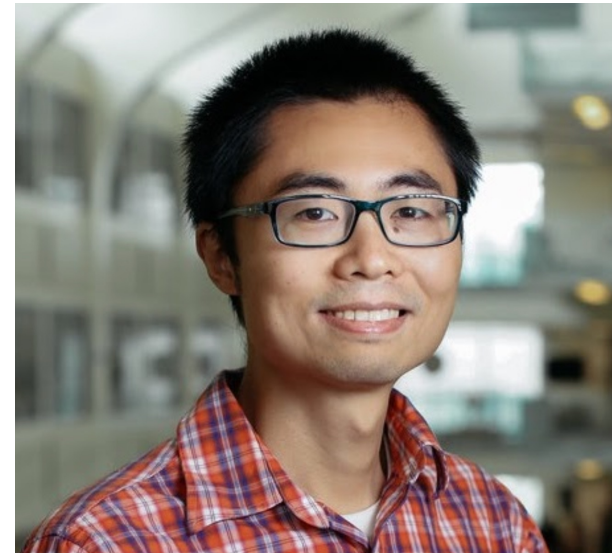


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# Collaborators

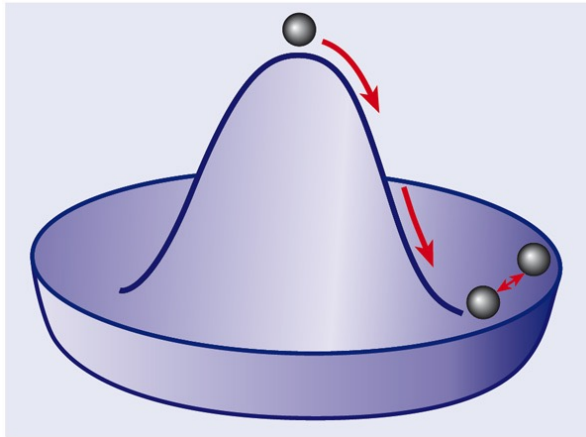


Yan-Qi Wang (UC Berkeley)



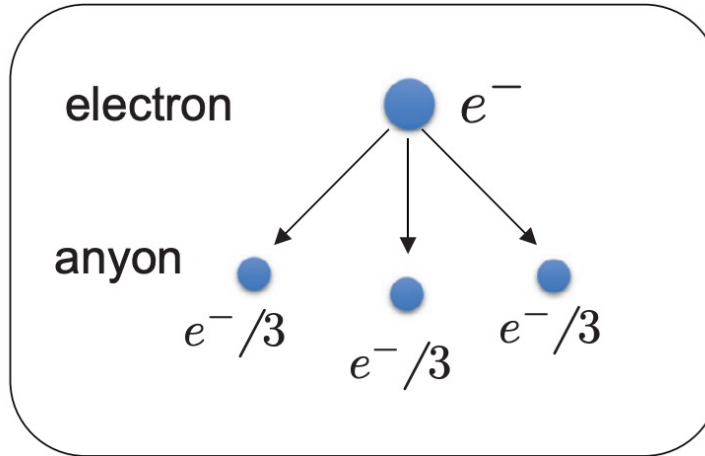
Yuanming Lu (Ohio State U)

*Theory of topological defects and textures in two-dimensional quantum orders with spontaneous symmetry breaking, Wang, CL, Lu, [arXiv:2211.13207](https://arxiv.org/abs/2211.13207)*



SSB:  $G \rightarrow H$

+



FQHE

= ?

In quantum Hall ferromagnet:

FQHE + SO(3)  $\rightarrow$  U(1)

**Skyrmions and the crossover from the integer to fractional quantum Hall effect  
at small Zeeman energies**

S. L. Sondhi,\* A. Karlhede,<sup>†</sup> and S. A. Kivelson

*Department of Physics, University of California, Los Angeles, Los Angeles, California 90024*

E. H. Rezayi

*Department of Physics and Astronomy, California State University, Los Angeles, Los Angeles, California 90032*

(Received 10 September 1992; revised manuscript received 24 May 1993)

solitons (skyrmions) on all length scales.<sup>22</sup> By virtue of the equality of the skyrmion density and the physical density it follows that the skyrmions carry charge  $\pm e$  according to the sense of the spin twist. This connection

Q: a general theory?

# Symmetry-breaking defects and textures in quantum orders

	H-symmetry enriched topological order (SET)	H-symmetry protected topological phase (SPT)
Topological defect	?	?
Topological texture	?	?

# Orders: comparison in 2+1D

	Classical order	Quantum order
Phases	Symmetry-breaking states	Topological order ( $\supset$ SPT+SET)
Wave function structure	Long-range correlation	Long-range entanglement
Examples	Néel order, CDW, Superfluid, SC...	IQHE, FQHE, Quantum spin liquid...
Theory	Landau theory for SSB: $G \rightarrow H$	TQFT Theory for anyons: $A$
Gapless excitations	Goldstone modes (SSB of continuous symmetry)	Photons (continuous gauge field)
Gapped excitations	Vortices, Skyrmions, Monopoles...	Anyons/bulk excitations

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Gapped excitations	Vortices, Skyrmions, Monopoles...	Anyons/bulk excitations

We assume the classical gapless Goldstone modes do not affect the gapped excitations (topological data).

# Classification of topological defects

	d = 1	d = 2	d = 3
$\pi_0$	Point defect	Line defect	Membrane defect
$\pi_1$	Texture (spacetime defect)	Point defect	Line defect
$\pi_2$		Texture (spacetime defect)	Point defect

	d=1	d=2	d=3
D=0			
D=1			
D=2			

Teo, Kane, PRB (2010)

# The topological theory of defects in ordered media\* †

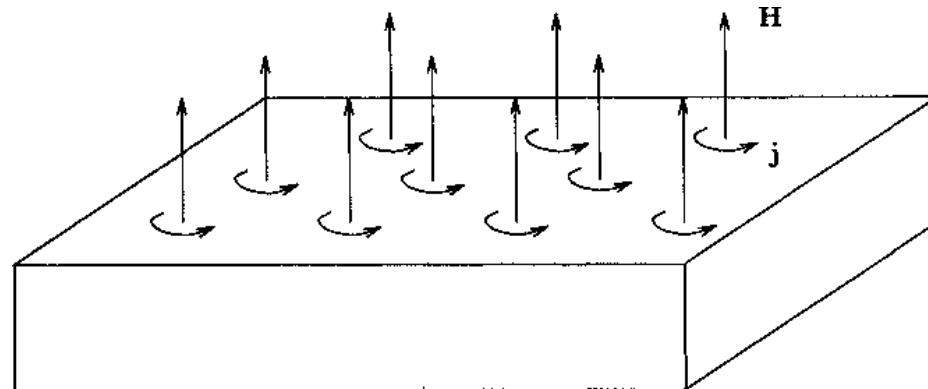
N. D. Mermin

*Laboratory of Atomic and Solid State Physics Cornell University, Ithaca, New York 14853*

Aspects of the theory of homotopy groups are described in a mathematical style closer to that of condensed matter physics than that of topology. The aim is to make more readily accessible to physicists the recent applications of homotopy theory to the study of defects in ordered media. Although many physical examples are woven into the development of the subject, the focus is on mathematical pedagogy rather than on a systematic review of applications.

$$\pi_1(S_1) = \mathbb{Z} .$$

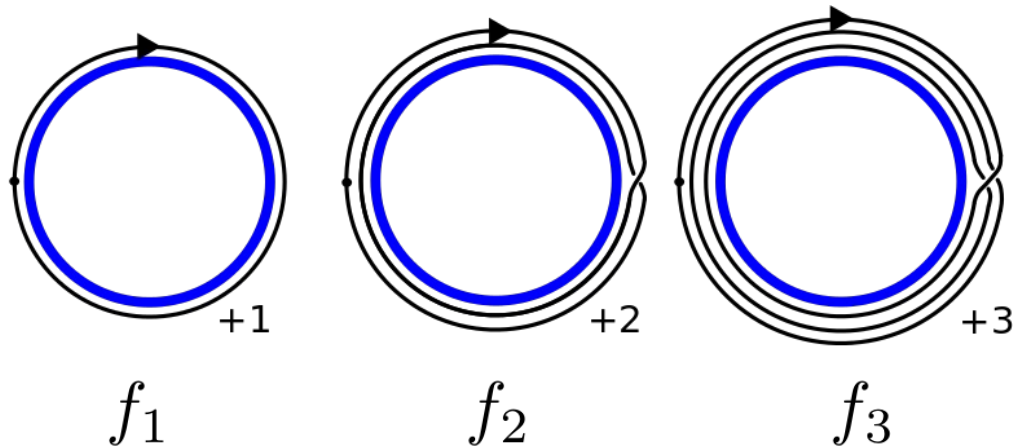
Mermin, RMP (1979)



# Understanding $\pi_1(S^1) = \mathbb{Z}$ .

$$\pi_1(S^1) = \{[f_0], [f_{+1}], [f_{-1}], \dots\} \cong \mathbb{Z}$$

$$f_n: S^1 \xrightarrow{\text{wind } n \text{ times}} S^1$$



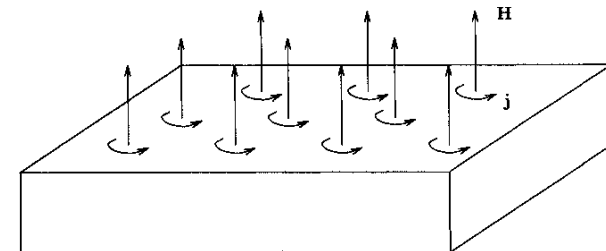
$$\text{SSB: } G \rightarrow H$$

$$\pi_1(G/H)$$

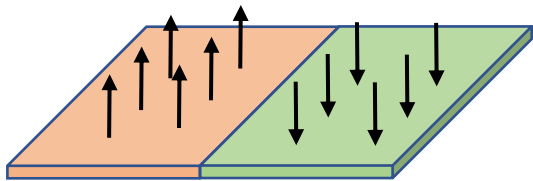
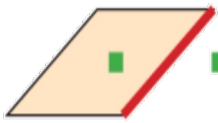
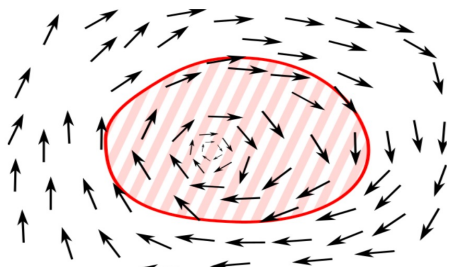
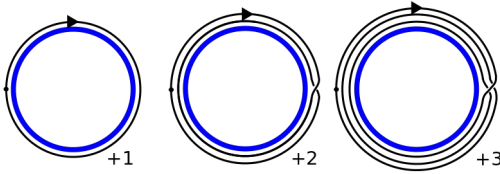
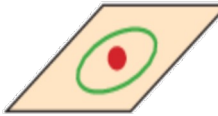
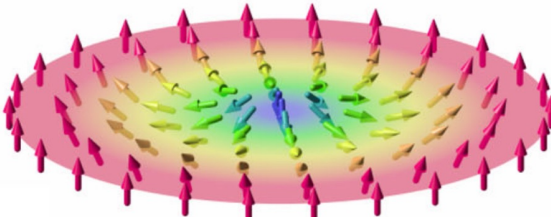
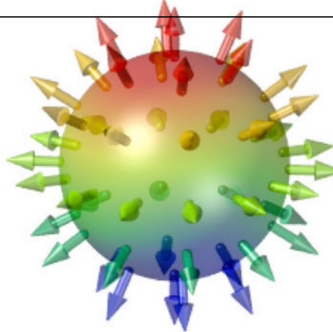
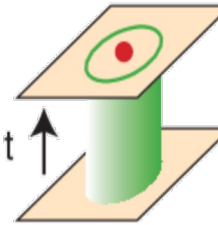
$$f: S^1 \rightarrow G/H$$

In type-II superconductor:

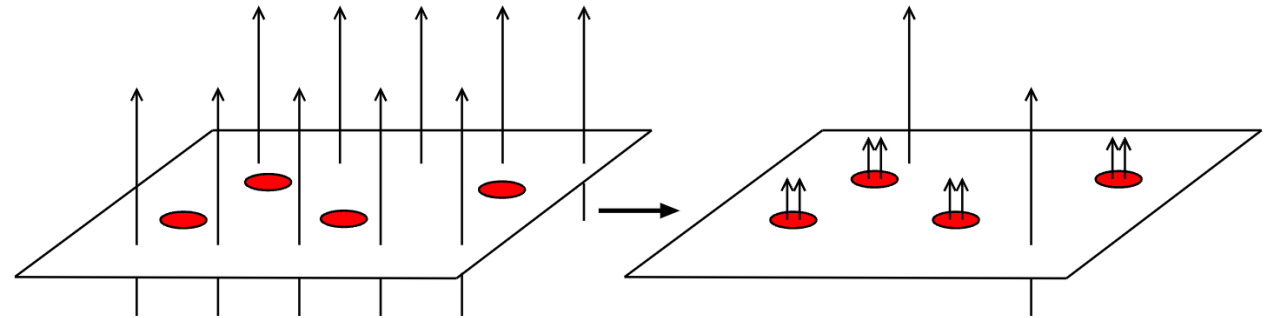
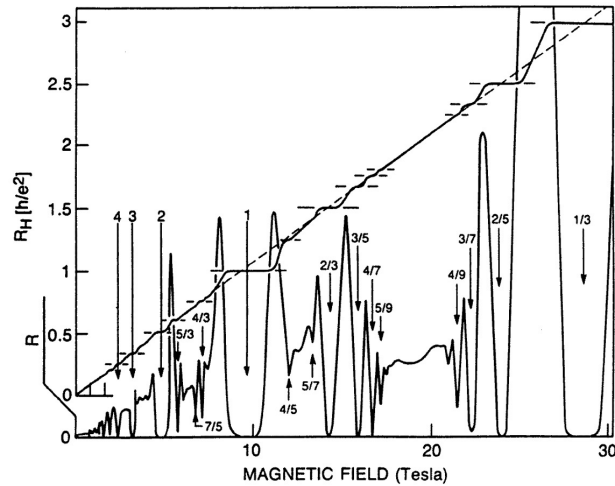
$$G = U(1), H = \mathbb{Z}_2, G/H = S^1$$



# Homotopy theory for topological excitations of an SSB state

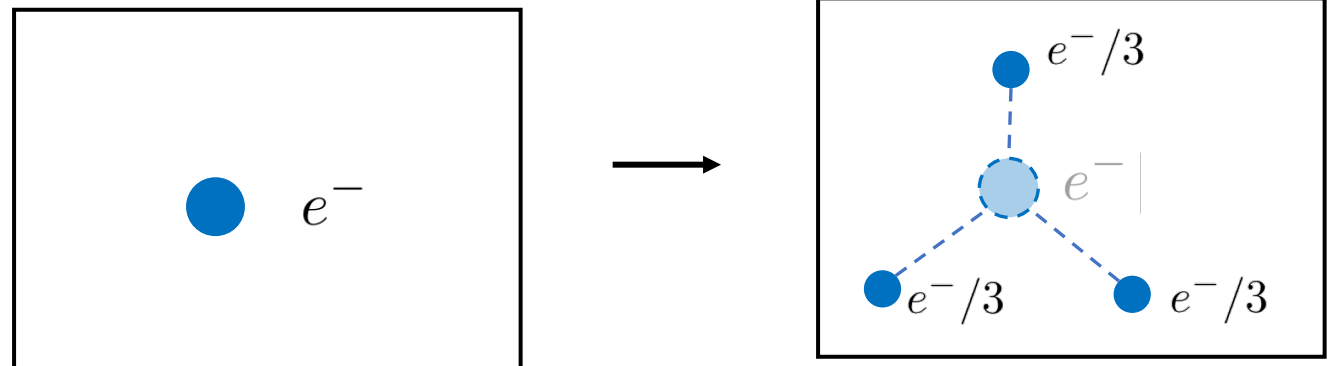
		<b>d = 2</b>				<b>d = 2</b>	
$\pi_0(G/H)$	Line defect	 <p>Domain wall</p>	$Z_2 \rightarrow \{1\}$ $G/H = Z_2$ $\pi_0(Z_2) = \mathbb{Z}_2$	$\{\uparrow, \downarrow\}$		D=0	
$\pi_1(G/H)$	Point defect	 <p>vortex</p>	$U(1) \rightarrow Z_2$ $G/H = S^1$ $\pi_1(S^1) = \mathbb{Z}$			D=1	
$\pi_2(G/H)$	Texture (spacetime defect)	 <p>Skyrmion</p>	$SO(3) \rightarrow SO(2)$ $G/H = S^2$ $\pi_2(S^2) = \mathbb{Z}$			D=2	

# Topological order: FQHE




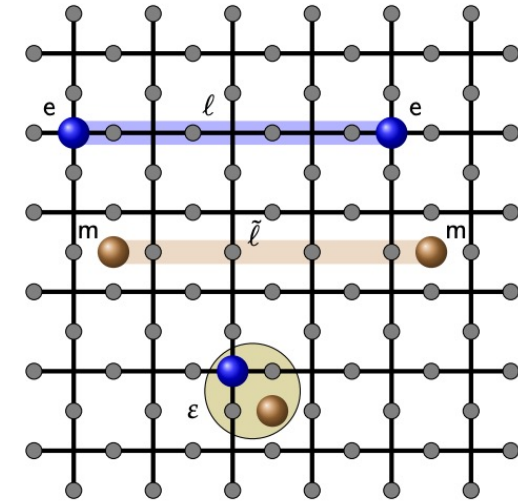
Topological order in 2+1D:

- Anyon types
- Anyon fusion
- Anyon braiding



# Topological order: toric code

$$H_{\text{tc}} = - \sum_{\text{square}} Z_1 Z_2 Z_3 Z_4 - \sum_{\text{star}} X_1 X_2 X_3 X_4$$




Spins are defined on the link

Topological order in 2+1D:

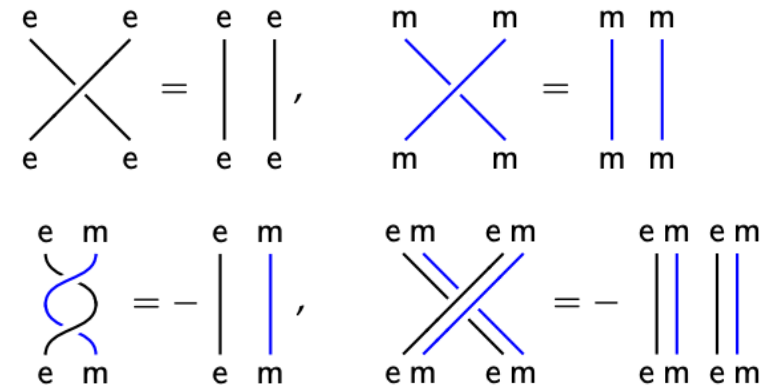
- Anyon types  $\mathcal{A} = \{1, e, m, \varepsilon\}$
- Anyon fusion
- Anyon braiding

$$e \times e = 1$$

$$m \times m = 1$$

$$\varepsilon \times \varepsilon = 1$$

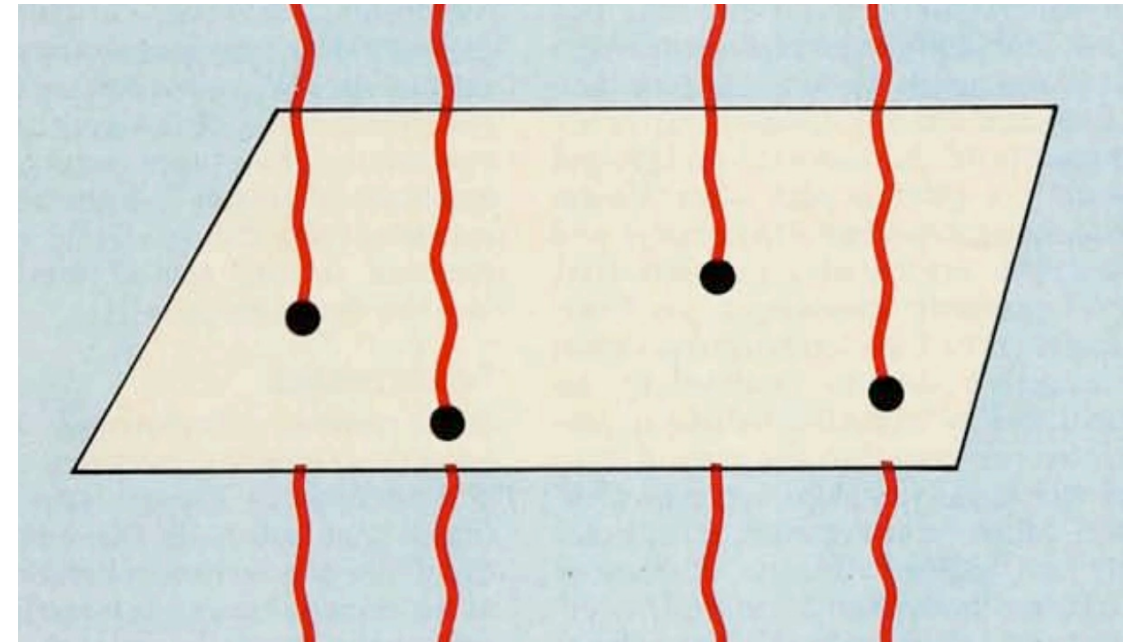
$$e \times m = \varepsilon$$



# Topological order in 2+1D

Examples	Anyon types	Anyon fusion	Anyon braiding
$\nu=1/3$ FQHE	$A = \langle e^{-}/3 \rangle$	$A \cong \mathbb{Z}_3$	Charge-flux
Toric code	$A = \{1, e, m, \varepsilon\}$	$A \cong \mathbb{Z}_2 \times \mathbb{Z}_2$	Boson (e,m) & Fermion ( $\varepsilon$ )

*Anyons, strings, gauge field, entanglement*



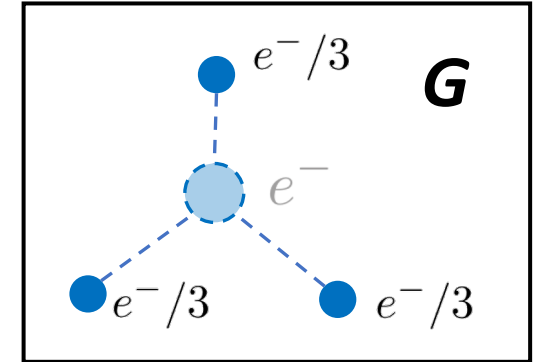
**Anyons** may be conceived of as particles carrying both electric charge and magnetic flux (red strings). The magnetic field is zero everywhere in the plane except at the location of the particles, and it causes no direct, classical interaction between the particles: Its role is to change the many-anyon wavefunction by a phase factor every time an anyon moves around another. The phase change is given by the Aharonov–Bohm effect.

# Topological order with symmetry

Q: Topological order in 2+1D

- Anyon types  $\mathcal{A}$
- Anyon fusion
- Anyon braiding

+ Symmetry  $\mathbf{G}$  ?



A: =

**G-Symmetry Enriched Topological phase**  
**(G-SET)**

Classification of G-SET:  $\mathcal{H}_\rho^2(G, \mathcal{A})$  the "H-upper-two"

# Understanding $\mathcal{H}_\rho^2(G, \mathcal{A})$

- Symmetry acts on the quantum state:  $g: |\psi\rangle \rightarrow U_g|\psi\rangle$

- No reason to expect the representation is faithful!

$$U_g U_h |\psi\rangle = e^{i\omega(g,h)} U_{gh} |\psi\rangle$$

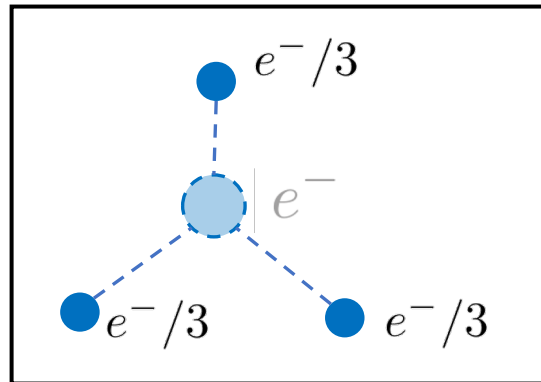
- Projective representation, classified by “H-upper-two”:  $\omega \in \mathcal{H}^2(G, U(1))$

- For SET, universal data all carried by anyons, hence:  $\omega \in \mathcal{H}_\rho^2(G, \mathcal{A})$

- Anyon permutation is faithful:  $\rho: G \rightarrow \text{perm}(\mathcal{A})$        $\omega: G \times G \rightarrow \mathcal{A}$

# Symmetry fractionalization and $\mathcal{H}_\rho^2(G, \mathcal{A})$

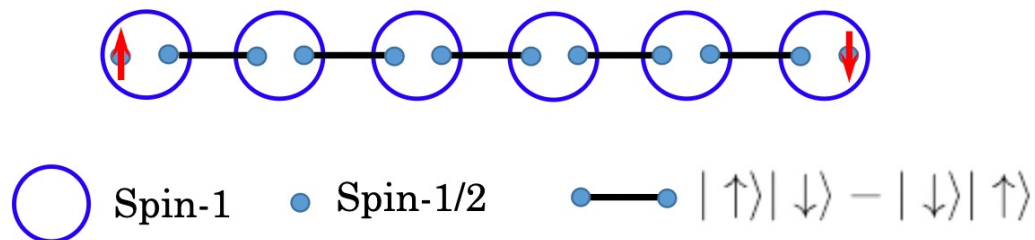
- Each anyon carries fractional quantum numbers of the symmetry



$$\mathcal{H}^2(U(1), \mathbb{Z}_3) = \mathbb{Z}_3$$

H-upper-2 always is an abelian group.

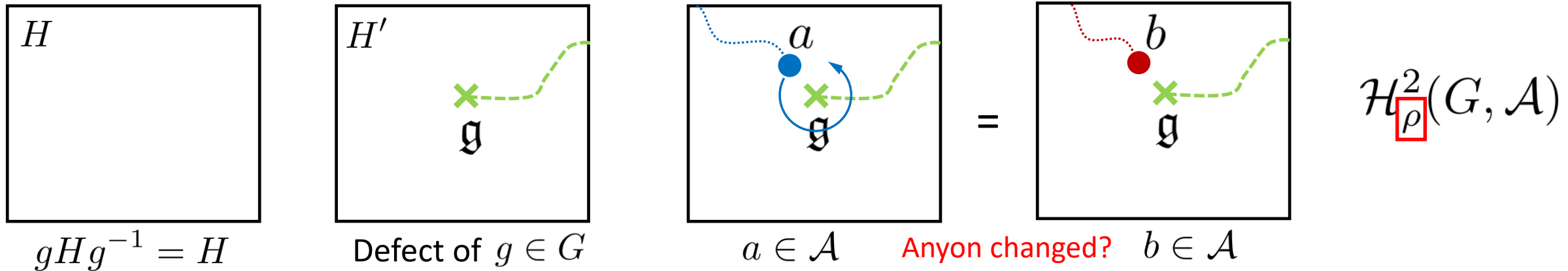
- A more familiar case: Haldane chain  $S = 1$ ,  $G = SO(3)$



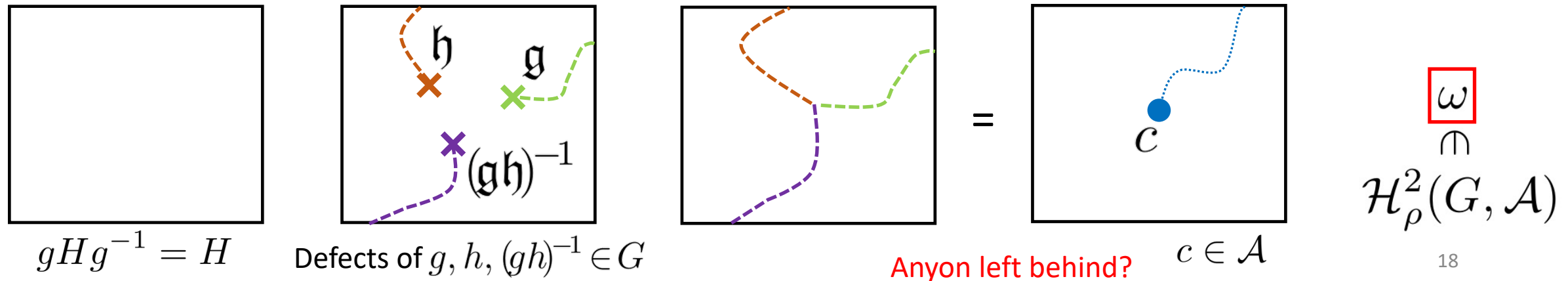
$$\mathcal{H}^2(SO(3), U(1)) = \mathbb{Z}_2$$

# Symmetry defect/flux and the A-B experiment

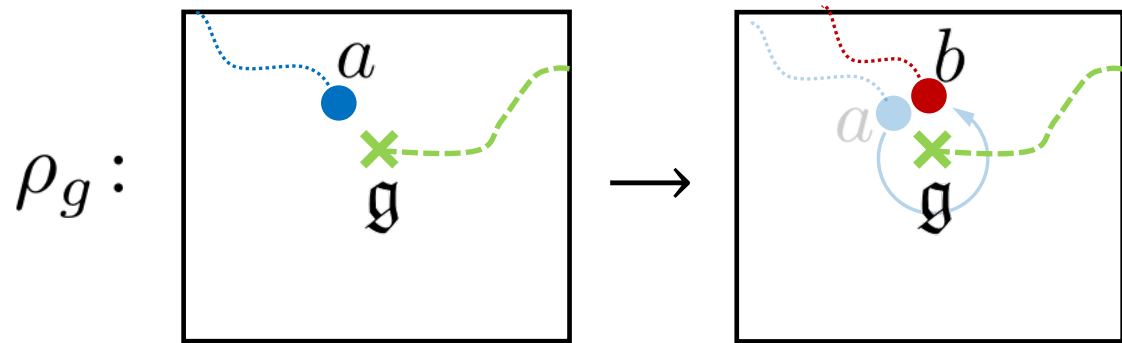
- Q1: How to probe anyon permutation by  $G$ ?



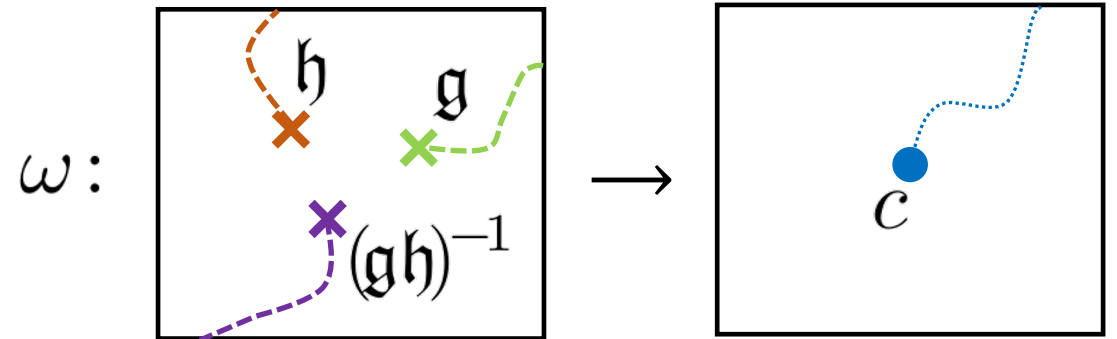
- Q2: How to determine the projective representation carried by the anyons?



# Symmetry defect/flux in $G$ -SET



$$\rho_g: a \mapsto b$$



$$\omega(g, h) = c \in \mathcal{A}$$

$$\omega \in \mathcal{H}_\rho^2(G, \mathcal{A})$$

## Symmetry defects

- are **extrinsic**: manually introduced, cost infinite energy
- have **strings attached**: symmetry broken only on a submanifold
- are **generalized anyons**: have their own fusion and braiding rules

*They are different from the topological defects associated with SSB.*

**Q:** Can we generalize the above theory to include **topological defects**?

# Our results: *Symmetry-breaking defects and textures in quantum orders*

Wang, CL, Lu, arXiv:2211.13207

Quantum orders

	<b>H-symmetry enriched topological order (SET)</b>	<b>H-symmetry protected topological phase (SPT)</b>
SSB {	Topological defect <ul style="list-style-type: none"><li>• Resembles symmetry flux</li><li>• Permute anyons</li><li>• Defect fractionalization</li></ul>	<ul style="list-style-type: none"><li>• Carries projective representation</li></ul>
	Topological texture <ul style="list-style-type: none"><li>• Carries symmetry charge</li></ul>	<ul style="list-style-type: none"><li>• Carries symmetry charge</li></ul>

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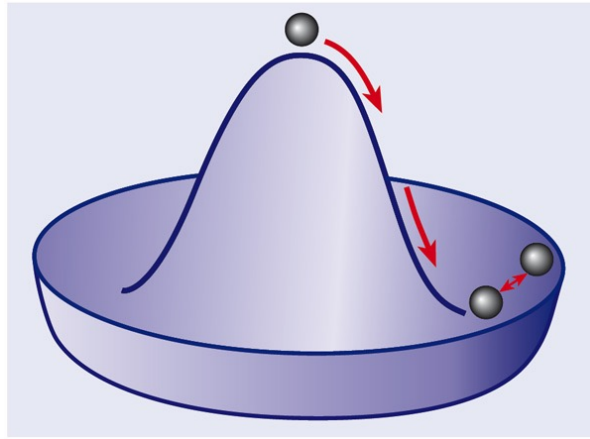
Quantum orders

		Quantum orders	
		H-symmetry enriched topological order (SET)	H-symmetry protected topological phase (SPT)
SSB	Topological defect	<ul style="list-style-type: none"> <li>Resembles symmetry flux</li> <li>Permute anyons</li> <li>Defect fractionalization</li> </ul> <p><b>Part I</b> "the correspondence"</p>	<ul style="list-style-type: none"> <li>Carries projective representation</li> </ul> <p><b>Part II</b> In the context of DQCP</p>
	Topological texture	<ul style="list-style-type: none"> <li>Carries symmetry charge</li> </ul>	<ul style="list-style-type: none"> <li>Carries symmetry charge</li> </ul>

# Part I

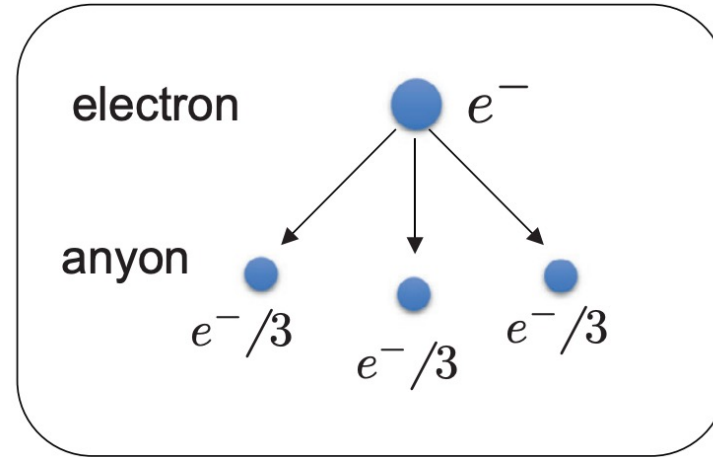
## Topological defects in SET

# $H$ -SET as a broken $G$ -symmetry phase



SSB:  $G \rightarrow H$

+



$H$ -SET

= ?

- Topological defect/vortex [classified by  $\pi_1(G/H)$ ] is present.
- Q: what can we say about these defects? In particular:
  - Can vortices permute anyons?
  - Can vortices fuse into anyons?
  - Relation between these vortices and the  $H$ -symmetry fluxes?

# Our claim

There is a map from the **fractionalization of symmetry flux** to the **fractionalization of symmetry-breaking defect**:

$$p^* : \mathcal{H}_\rho^2(H, \mathcal{A}) \rightarrow \mathcal{H}_{\tilde{\rho}}^2(\pi_1(G/H), \mathcal{A})$$

The realizable ones of the **latter** in the  $H$ -SET are

$$\text{im}(p^*)$$

# Main results

- Result 1: example calculations

No.	$G$	$H$	$H$ -action $\rho$	$\mathcal{H}_\rho^2(H, \mathcal{A})$	$\rightarrow$	$\mathcal{H}_{\tilde{\rho}}^2(\pi_1(G/H), \mathcal{A})$
1	$U(1)$	$Z_2$	Trivial	$\mathbb{Z}_2^e \times \mathbb{Z}_2^m$	$\rightarrow$	0
2	$U(1)$	$Z_2$	Nontrivial	0	$\rightarrow$	0
3	$SO(3)$	$D_2$	Trivial	$(\mathbb{Z}_2^3)^e \times (\mathbb{Z}_2^3)^m$	$\xrightarrow{\text{surj.}}$	$(\mathbb{Z}_2^2)^e \times (\mathbb{Z}_2^2)^m$
4	$SO(3)$	$D_2$	Nontrivial	$\mathbb{Z}_2^\varepsilon$	$\xrightarrow{0}$	$\mathbb{Z}_2^\varepsilon$

- Result 2: a theorem

If  $\pi_1(G/H)$  is of a semi-direct product form  $\pi_1(G/H) \cong \pi_1(G) \rtimes H$ , then

$$\mathcal{H}_\rho^2(H, \mathcal{A}) \xrightarrow{\cong} \mathcal{H}_{\tilde{\rho}}^2(\pi_1(G/H), \mathcal{A}).$$

- Result 3: nontrivial fusion of dislocations in toric code

$$\mathcal{H}^2(\mathbb{Z}^2, \mathbb{Z}_2^m) \cong \mathcal{H}^2(\pi_1(G/H), \mathbb{Z}_2^m) = \mathbb{Z}_2$$

# Main results

- Result 1: example calculations

No.	$G$	$H$	$H$ -action $\rho$	$\mathcal{H}_\rho^2(H, \mathcal{A})$	$\rightarrow$	$\mathcal{H}_{\tilde{\rho}}^2(\pi_1(G/H), \mathcal{A})$	
1	$U(1)$	$Z_2$	Trivial	$\mathbb{Z}_2^e \times \mathbb{Z}_2^m$	$\rightarrow$	$0$	
2	$U(1)$	$Z_2$	Nontrivial	$0$	$\rightarrow$	$0$	No-go!
3	$SO(3)$	$D_2$	Trivial	$(\mathbb{Z}_2^3)^e \times (\mathbb{Z}_2^3)^m$	$\xrightarrow{\text{surj.}}$	$(\mathbb{Z}_2^2)^e \times (\mathbb{Z}_2^2)^m$	Nothing beyond!
4	$SO(3)$	$D_2$	Nontrivial	$\mathbb{Z}_2^\varepsilon$	$\xrightarrow{0}$	$\mathbb{Z}_2^\varepsilon$	No-go!

$\mathcal{A} = \{1, e, m, \varepsilon\}$

- Result 2: a theorem

If  $\pi_1(G/H)$  is of a semi-direct product form  $\pi_1(G/H) \cong \pi_1(G) \rtimes H$ , then

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- Result 3: nontrivial fusion of dislocations in toric code

$$\mathcal{H}^2(\mathbb{Z}^2, \mathbb{Z}_2^m) \cong \mathcal{H}^2(\pi_1(G/H), \mathbb{Z}_2^m) = \mathbb{Z}_2$$

# Result 1: pair superfluid and biaxial nematics

	No.	$G$	$H$	$H$ -action $\rho$	$\mathcal{H}_\rho^2(H, \mathcal{A})$	$\rightarrow$	$\mathcal{H}_\rho^2(\pi_1(G/H), \mathcal{A})$
Pair superfluid	1	$U(1)$	$Z_2$	Trivial	$\mathbb{Z}_2^e \times \mathbb{Z}_2^m$	$\rightarrow$	0
	2	$U(1)$	$Z_2$	Nontrivial	0	$\rightarrow$	0
Biaxial nematics	3	$SO(3)$	$D_2$	Trivial	$(\mathbb{Z}_2^3)^e \times (\mathbb{Z}_2^3)^m$	$\xrightarrow{\text{surj.}}$	$(\mathbb{Z}_2^2)^e \times (\mathbb{Z}_2^2)^m$
	4	$SO(3)$	$D_2$	Nontrivial	$\mathbb{Z}_2^\varepsilon$	$\xrightarrow{0}$	$\mathbb{Z}_2^\varepsilon$

For 3, 4:  $\pi_1(G/H) = Q_8$

Model constructions:

$$\hat{H} = \hat{H}_{\text{TO}}[\hat{\chi}, \hat{f}] + \hat{H}_{\text{SSB}}[\hat{S}] + \hat{H}_{\text{int}}[\hat{\chi}, \hat{f}, \hat{S}]$$

Majorana fermion

complex fermion

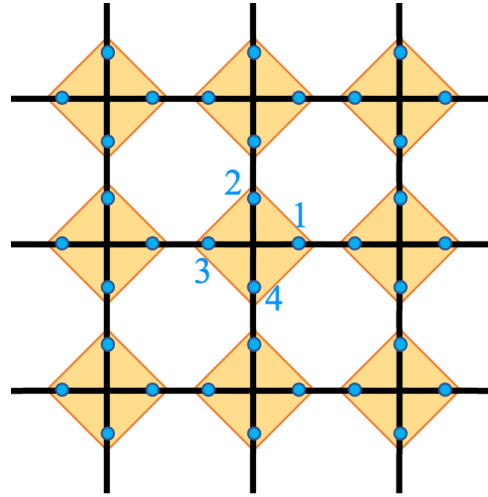
spin

# Result 1: pair superfluid and biaxial nematics

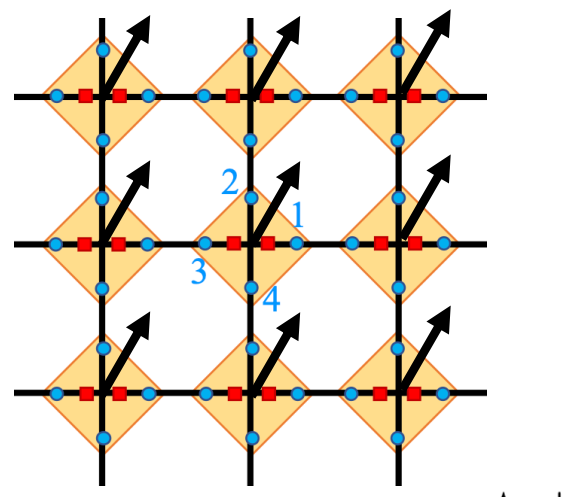
$$\hat{H} = \hat{H}_{\text{TO}}[\hat{\chi}, \hat{f}_\sigma] + \hat{H}_{\text{SSB}}[\hat{S}] + \hat{H}_{\text{int}}[\hat{\chi}, \hat{f}_\sigma, \hat{S}]$$

●   ■
↗
●   ■
↗

- **SSB:** Bose-Hubbard-like model for  $S$
- **Topological order:** Majorana ( $\chi$ ) construction for toric code
- **Interaction:** p+ip superconductor for the complex fermion ( $f$ ), couple to both  $\chi$  and  $S$

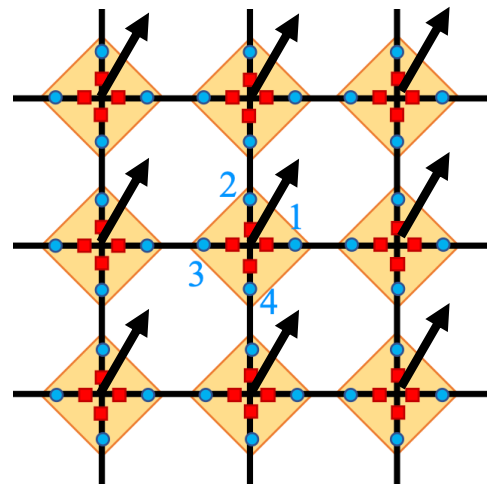


Toric code



■  $\sigma = \uparrow, \downarrow$

Pair superfluid



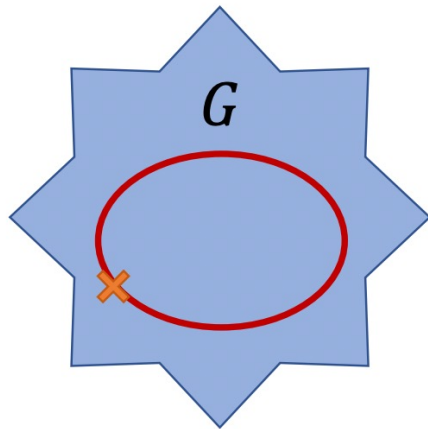
■  $\sigma = s, p_x, p_y, p_z$

Biaxial nematics

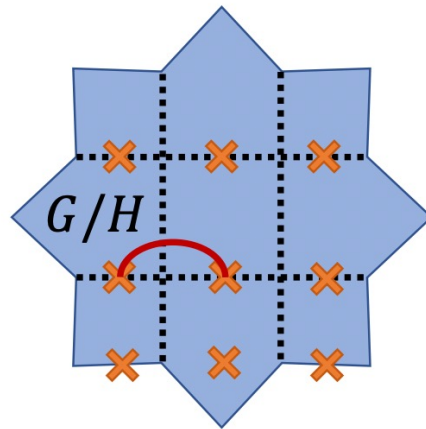
# Result 2: a theorem

If  $\pi_1(G/H)$  is of a semi-direct product form  $\pi_1(G/H) \cong \pi_1(G) \rtimes H$ , then

$$\mathcal{H}_\rho^2(H, \mathcal{A}) \xrightarrow{\cong} \mathcal{H}_\rho^2(\pi_1(G/H), \mathcal{A}).$$



$\pi_1(G)$

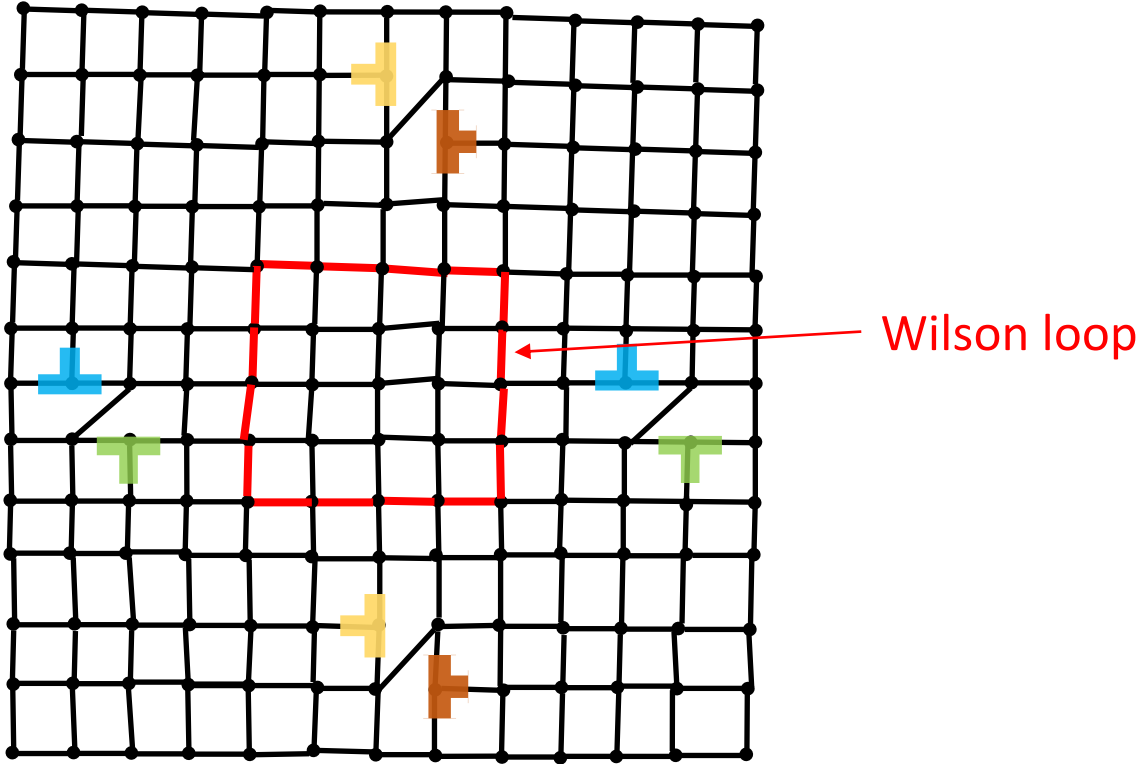
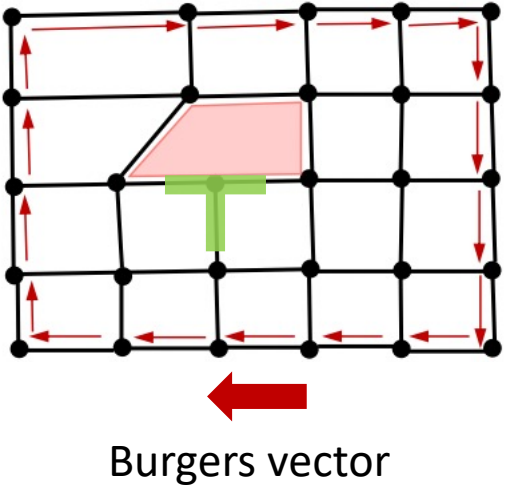


$\pi_1(G/H)$

$$\pi_1(G) \triangleleft \pi_1(G/H),$$

$$\pi_1(G/H)/\pi_1(G) \cong H$$

# Result 3: fusion of dislocations in toric code



$$R^2 \rightarrow Z^2$$

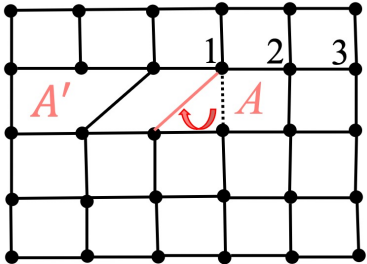
$$\pi_1(G/H) = \mathbb{Z} \times \mathbb{Z}$$

Mermin, RMP (1979)

$$\mathcal{H}^2(\mathbb{Z}^2, \mathbb{Z}_2^m) \cong \mathcal{H}^2(\pi_1(G/H), \mathbb{Z}_2^m) = \mathbb{Z}_2$$

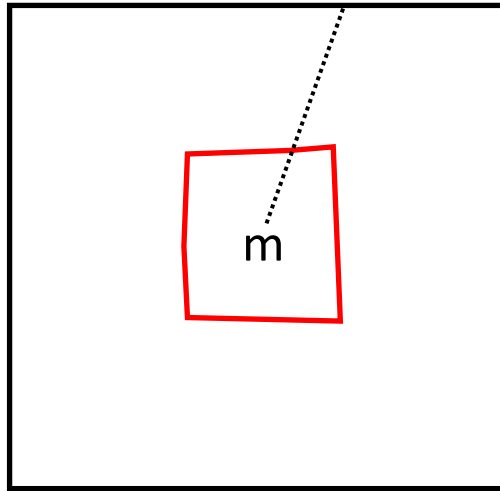
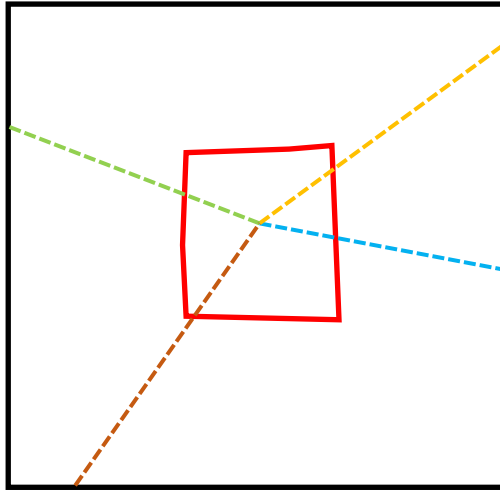
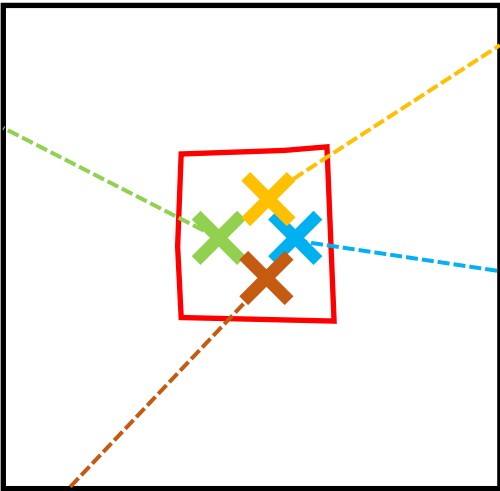
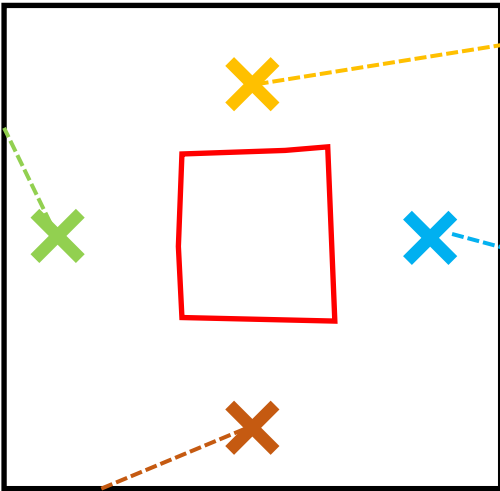
$$T_x T_y T_x^{-1} T_y^{-1} = -1$$

# Result 3: fusion of dislocations in toric code



$$U_{T_x^+}(v) = \frac{1 + Z_{v, v-\hat{e}_x}}{2} + \frac{1 - Z_{v, v-\hat{e}_x}}{2} X_{v, v+\hat{e}_y}$$

$$U_{T_x^+}(v) Z_{v, v-\hat{e}_y} U_{T_x^+}^{-1}(v) = Z_{v-\hat{e}_x-\hat{e}_y, v} Z_{v-\hat{e}_y, v-\hat{e}_x-\hat{e}_y}$$

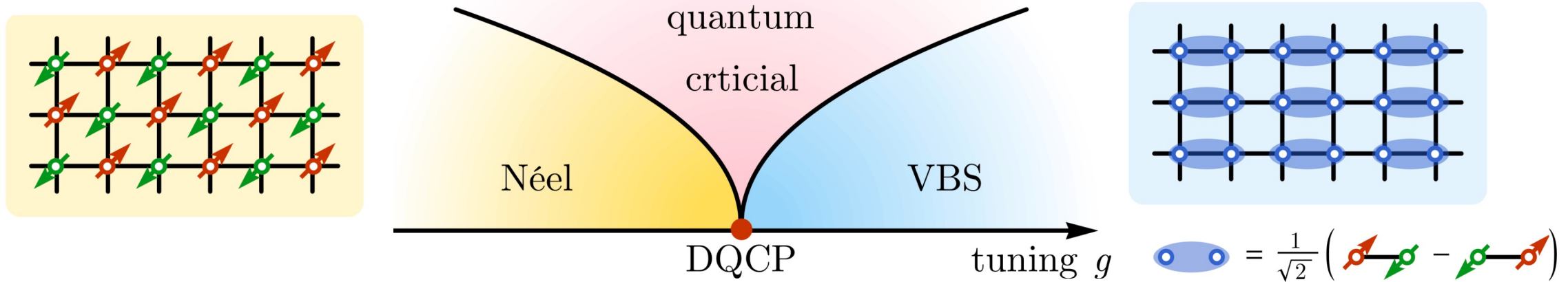


Translation flux are in one-to-one correspondence with dislocations:  $\mathcal{H}^2(\mathbb{Z}^2, \mathbb{Z}_2^m) \cong \mathcal{H}^2(\pi_1(G/H), \mathbb{Z}_2^m) = \mathbb{Z}_2$

## Part II

# Topological defects and textures in SPT: Understanding of DQCP

# Deconfined quantum critical point (DQCP)

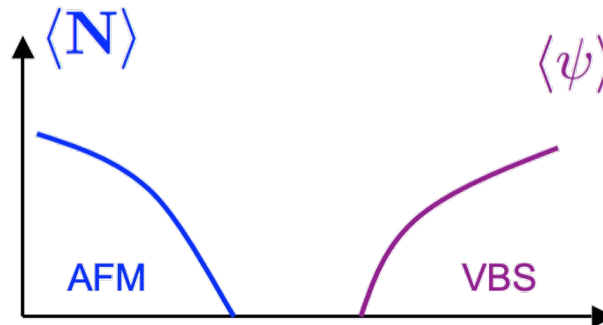


$$H_{\text{Néel}} = C_4 \times U(1)_{S^z}$$

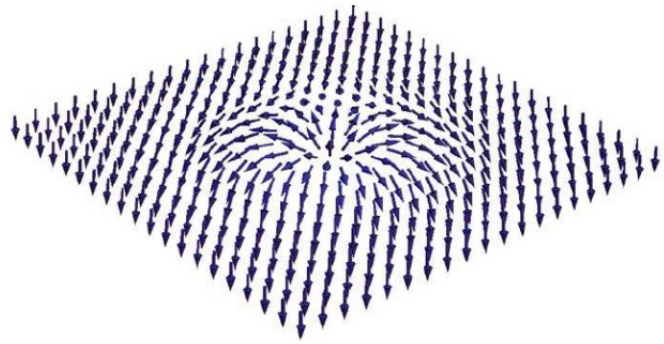
$$G = C_4 \times SO(3) \times Z_2^T$$

$$H_{\text{VBS}} = SO(3) \times Z_2^T$$

Landau-Ginzburg-Wilson prediction:



# Defect/texture picture for DQCP

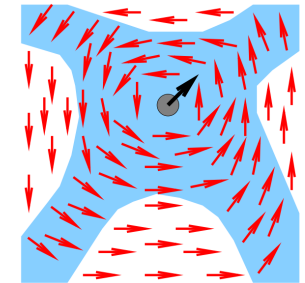
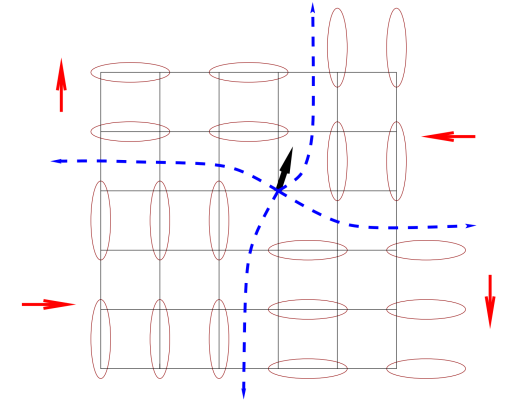
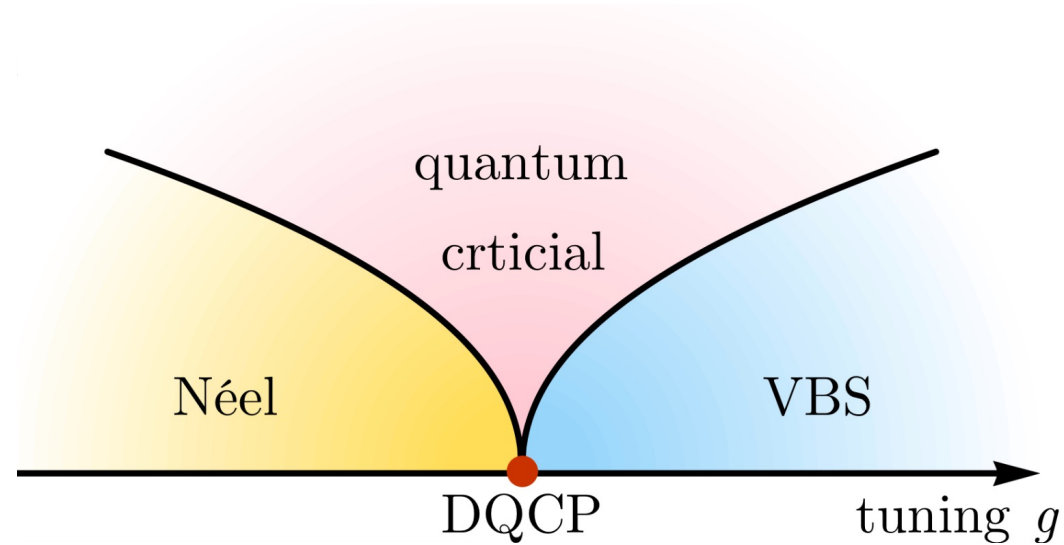


1	$i$	1	$i$
$-i$	-1	$-i$	-1
1	$i$	1	$i$
$-i$	-1	$-i$	-1

Picture from the Néel order:  
Skyrmion carries  $C_4$  angular momentum

$$(n^1, n^2, n^3)$$

$$\pi_2(G/H_{\text{Néel}}) = \pi_2(S^2) = \mathbb{Z}$$



Picture from the VBS order:  
Vortex core carries a spin 1/2

$$(n^4, n^5)$$

$$\pi_1(\tilde{G}/H_{\text{VBS}}) = \pi_1(S^1) = \mathbb{Z}$$

$$S_{\text{WZW}}[\vec{n}]$$

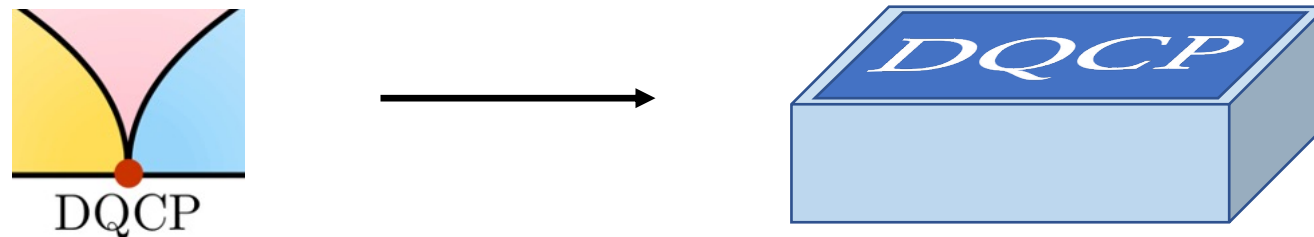
$$\vec{n} = (n^1, n^2, n^3, n^4, n^5)$$

# An SPT theory for DQCP

bSPT (Bosonic symmetry protected topological phases):

- Short-range entangled topological phases of bosons
- Input: bosonic system; symmetry  $G$
- Considering  $G \rightarrow$  distinct phases that reduces to the trivial one when ignoring  $G$
- “Anomalous” boundaries: boundary must be gapless
- Classification: in  $d$ -spatial dimension, “H-upper- $(d+1)$ ”

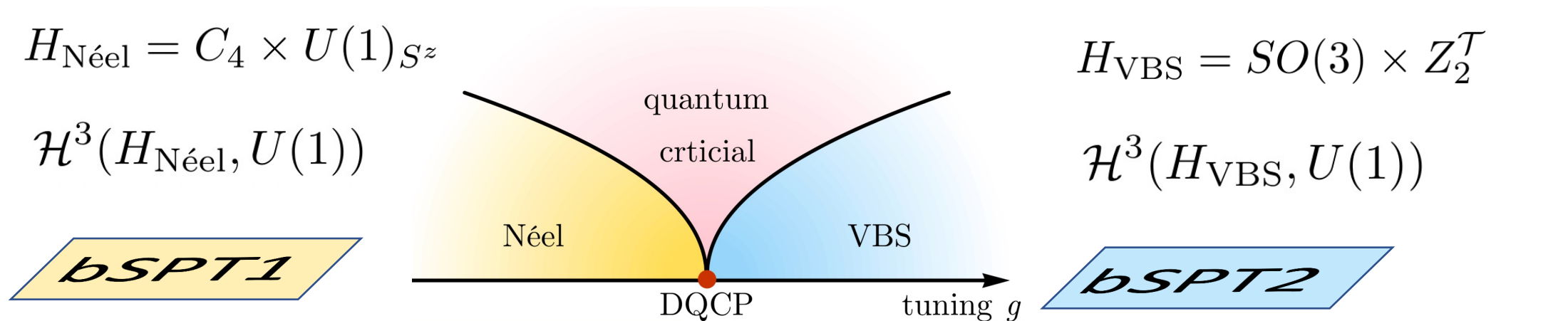
Point of view: viewing DQCP as the anomalous boundary of a 3+1D system (“H-upper-four”)



# Another SPT theory for DQCP?

A connection between DQCP and 2+1D SPT (“H-upper-three”):

DQCP can be “derived” from assuming both ordered states are bulk bSPTs



Skyrmion carries  $C_4$  angular momentum;  
Skyrmion is equivalent to  $2\pi$  flux;

$$\mathcal{H}^2(U(1)_{S^z}, \mathcal{H}^1(C_4, U(1))) \subset \mathcal{H}^3(H_{\text{Néel}}, U(1))$$

$$\mathcal{H}^2(U(1), M) = M = \mathcal{H}^1(\pi_1(U(1)) = \mathbb{Z}, M)$$

Vortex carries a spin 1/2

$$\mathcal{H}^1(\pi_1(SO(2)), \mathcal{H}^2(H_{\text{VBS}}, U(1)))$$

$$\subset \mathcal{H}^3(\tilde{H}_{\text{VBS}}, U(1))$$

# DQCP from SPT: what are we claiming?

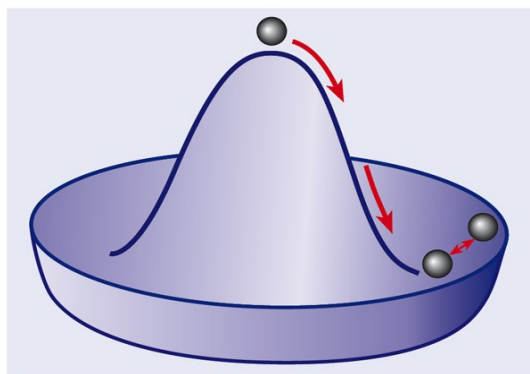
- We are claiming that the physics for DQCP predicted from the field theory side can be understood from the algebraical data of bSPT.
- The algebraic data here are defect, texture, the quantum number and projective representations they can carry. These are the minimal data.
- The Kunneth decomposition provides a concise way to see the mixed anomaly. It is completely equivalent to the field theory approach.
- While we rationalized the DQCP between Néel-VBS in our algebraic formulism, the goal is to predict more DQCPs in systems with other symmetries.
- Other examples of DQCP: between QSH insulator and pair superfluid.

# Summary and discussion

- Correspondence between SSB defect and symmetry flux in SET

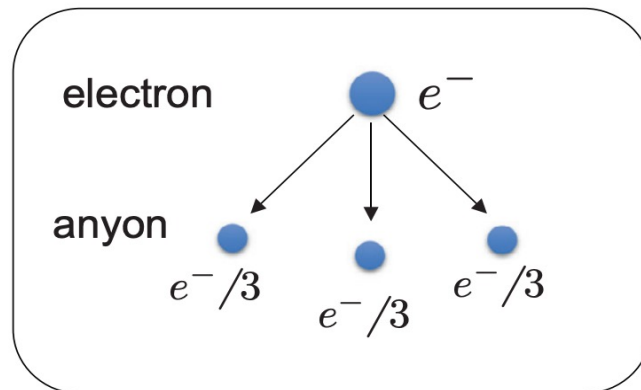
$$\text{im} \left( p^* : \mathcal{H}_\rho^2(H, \mathcal{A}) \rightarrow \mathcal{H}_\rho^2(\pi_1(G/H), \mathcal{A}) \right)$$

- SSB defect can permute anyons;
  - SSB defect fuse to anyons;
  - Explicit models: toric code with pair superfluid/biaxial nematics/dislocation
- SSB defects and textures in SPT and DQCP
    - Assumption: both orders are SPT
    - the Néel side: skyrmion carries symmetry quantum number
    - The VBS side: vortex carries projective representation
- Capability and limitation of the algebraic approach
    - Universal (topological) data
    - No energy scale; no energetics



SSB:  $G \rightarrow H$

+

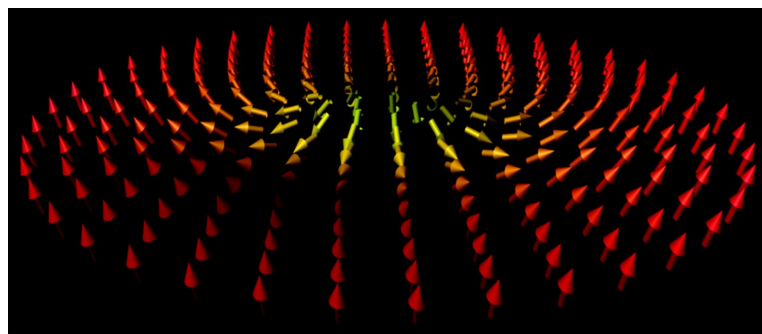


H-SET

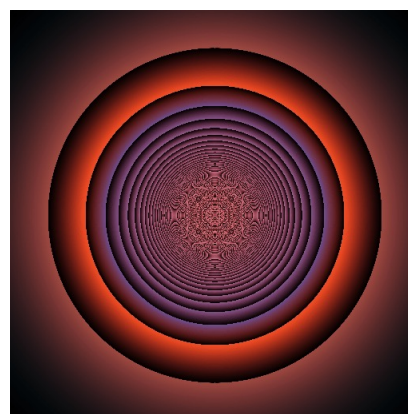
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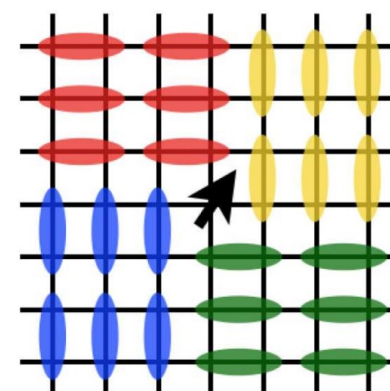
Thank you!



Skyrmion



DQCP



vortex